The switch has been in position a for a long time. At time $t = 0$ the switch moves to position b.

a) Find the initial conditions $v_C(0^+)$ and $i_L(0^+)$.  

b) Find $v(t)$ for $t \geq 0$.  

c) Find the characteristic equation that governs transient behavior, if $R = 200 \ \Omega$.  

d) Will the response be overdamped, underdamped, or critically damped?  

e) Repeat c)-d) for $R = 312.5 \ \Omega$ and $R = 250 \ \Omega$.  

KCL: \( i_C(t) + i_L(t) + i_R(t) = 0 \); \( i_C(t) = C \frac{dv_C}{dt} \), \( i_L(t) = \frac{1}{L} \int_0^t v(t) \, dt \)

KCL becomes: \( c \frac{dv}{dt} + \frac{1}{RC} \int v(t) \, dt + \frac{v(t)}{R} = 0 \). Taking derivative with respect to \( t \) yields

\[
\frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = 0, \quad v(0^+) = 10, \quad \frac{dv(0^+)}{dt} = -\frac{10}{RC}
\]

Assuming a solution of the form \( v(t) = e^{st}, \ t \geq 0 \), the characteristic equation is

\[
s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0
\]

The roots, denoted as \( s_1, s_2 \), are given by

\[
s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4RC^2} - \frac{1}{LC}}
\]

Three cases (\( L = 50 \ \text{mH}, C = 0.2 \ \mu F \))

a) Two different real-valued roots (over-damped solution)
   \( (R = 200 \ \Omega, s_1 = -5,000, s_2 = -20,000) \)

b) Repeated real-valued roots (critically damped solution)
   \( (R = 250 \ \Omega, s_1, s_2 = -10,000) \)

c) Two complex-valued roots (under-damped solution)
   \( (R = 312.5 \ \Omega, s_1 = -8,000 + j6,000, s_2 = -8,000 - j6,000) \)

Case a) The general solution is of the form

\[
v(t) = v_p(t) + A_1 e^{s_1 t} + A_2 e^{s_2 t}, \quad t \geq 0,
\]

where, in this case, \( v_p(t) = 0 \).

We use the initial conditions to solve for the constants \( A_1 \) and \( A_2 \). Evaluating at \( t = 0 \)

\[
v(0) = 10 = A_1 + A_2
\]

\[
\frac{dv(0)}{dt} = -\frac{10}{RC} = A_1 s_1 + A_2 s_2
\]

The equations for the unknown constants can be expressed as a vector equation

\[
\begin{bmatrix}
1 & 1 \\
1 & s_2
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix} =
\begin{bmatrix}
10 \\
-10/RC
\end{bmatrix}
\]

and easily solved, with \( A_1 = \frac{-10}{3}, \ A_2 = \frac{40}{3} \). The solution is then

\[
v(t) = \frac{-10}{3} e^{-5,000 t} + \frac{40}{3} e^{-20,000 t}, \quad t \geq 0.
\]

The other cases are solved in a similar manner.

Numerical evaluation of the solutions for cases a)-c) are shown below.
function lecture2example
% This function plots the over-damped, under-damped, and critically-damped voltage solutions to the
% circuit example presented in lecture 2.
% R = 200 (over-damped), 312.5 (under-damped), or 250 ohms (critically damped).
t=[0:0.00001:0.001]; % Range [0, 1ms], time step = 10^(-5) sec
vover=(-10/3)*exp(-5000*t) + (40/3)*exp(-20000*t); % over-damped solution
vunder=exp(-8000*t) .* (10*cos(6000*t) - (40/3)*sin(6000*t)); % under-damped
vcrit=(10 - 100000*t) .* exp(-10000*t); % critically-damped solution
figure(1)
plot(t,vover,t,vunder,'--',t,vcrit,'-.')
xlabel('Time, t, sec')
ylabel('Voltage, v(t), volts')
title('Over-, Under-, and Critically-Damped Solutions, Example, Lecture 2')
legend('Over-Damped', 'Under-Damped', 'Critically-Damped')