This course project is intended to be an individual effort project. The student is required to **complete the work individually, without help from anyone else.** (The student may, however, get help from the instructor or TA.) The student must turn in this page with the report, including signature on the pledge at the bottom. **The final project report is due at the beginning of class lecture on Wednesday, April 17.** Late reports may be turned in on Friday, April 19, for a 20% reduction in grade. The project tasks are outlined on the following pages. The student must produce a **typed report**, including equations, figures and tables, that includes discussion of the results and any conclusions reached. The report may contain a comparison of techniques, a discussion of the behavior of the circuit as a filter, and any positive outcomes of the efforts of the project. The guidelines for good technical report writing need to be followed. We are particularly interested in reports demonstrating abilities related to the following ABET Criteria 3 outcomes: (a) an ability to apply knowledge of mathematics, science, and engineering; (b) an ability to design and conduct experiments as well as analyze and interpret data; (g) an ability to communicate effectively; (k) an ability to use techniques, skills, and modern engineering tools necessary for engineering practice.

**Notices (standard School EE 321 project requirements):**
1) A preliminary report, consisting of Part I, tasks 1), 2) and 3), is due Wednesday, March 27, at the beginning of class lecture. For this preliminary due date, turn in only the results, the MATLAB commands, and a signed copy of the pledge below; no typed report is needed. Your results should present a derivation of the differential equation and the state space model. This preliminary report is worth 10 points out of the total 100 points on the project grade. You may turn in Part I on Friday, March 29 for a 20% reduction in that part of the grade (i.e., for a maximum of 8 points). The final report must also include all of Part I.
2) Any evidence of collaboration with other students on either the final project report or the preliminary results will result in a grade of zero for the project for all collaborators. Additionally, the violation will be reported to the School of EECS and the University.
3) Write the report in your own words; do not use sentences or paragraphs from any other source. Grading will be based, in part, on content and grammar, so be sure to proof read your report. If significant portions of the report are found to be copied from another source, without proper attribution, a grade of zero will be assigned for the project and the plagiarism reported to the School and University.
4) The pledge below must be signed and turned in with both the preliminary results and the final project report. Final project reports without the signed pledge will receive a grade of zero.

**PLEDGE:** I HAVE NOT OBTAINED ASSISTANCE IN COMPLETING THIS PROJECT FROM ANYONE OTHER THAN THE INSTRUCTOR OR TA FOR THIS COURSE, NOR HAVE I GIVEN ASSISTANCE TO OTHER MEMBERS OF THIS CLASS.

SIGNED: __________________________________________________________
A circuit with input voltage \( v_i(t) \), and output voltage \( v_o(t) \), is shown below, with complex impedances in the circuit representing either a resistor (\( Z(s) = R \)), an inductor (\( Z(s) = sL \)), or a capacitor (\( Z(s) = 1/sC \)), or some combination of any of the three.

![Generic Circuit](image)

Figure 1. Generic circuit with input voltage \( v_i(t) \), and output voltage \( v_o(t) \).

**Part 1.**

1. Use \( s \)-domain techniques and determine the transfer function

\[
H(s) = \frac{V_o(s)}{V_i(s)}
\]

in terms of the impedances \( Z_1, Z_2, Z_3, \) and \( Z_4 \).

Solution: Use KVL around the two loops, with \( I_1(s) \) and \( I_2(s) \) the loop currents.

\[
V_i(s) = I_1(Z_1 + Z_2) - I_2(Z_2)
\]

\[
0 = -I_1Z_2 + I_2(Z_2 + Z_3 + Z_4)
\]

Writing as a matrix equation and using Cramer’s rule yields

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2Z_4}{Z_1(Z_2 + Z_3 + Z_4) + Z_2(Z_3 + Z_4)}.
\]

Substituting for the component values yields the transfer function

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^3 + \left(\frac{1}{C_1R_1} + \frac{1}{C_2R_2} + \frac{1}{C_1R_1}\right)s^2 + \left(\frac{1}{C_1C_2R_1R_2} + \frac{1}{LC_1}\right)s + \frac{1}{LC_1C_2R_2}}.
\]

2. Let \( Z_1(s) = sL \), \( Z_2(s) = R_1||\frac{1}{sC_1} \), \( Z_3(s) = R_2 \), and \( Z_4(s) = 1/sC_2 \), corresponding to the circuit below. Using Kirchhoff’s voltage and currents laws, derive the 3\(^{rd} \) order differential equation for \( v_o(t) \). Assume that there is no energy stored in the capacitors or inductor at time \( t = 0^- \). From the differential equation, find a state variable representation, and specify the state variable matrices \( A, B, C, D \), to generate the voltages \( v_o(t) \) and \( v_L(t) \). From the differential equation, determine the transfer function, \( H(s) = \frac{V_o(s)}{V_i(s)} \). Verify that it is equivalent to the result in 1) for the choice of impedances in 2).
Approach: Use KVL and KCL, selecting as variables inductor currents and capacitor voltages.

KVL: \( v_i(t) = v_L(t) + v_{C1}(t) \)  
(1)

KVL: \( v_{C1}(t) = v_{R2}(t) + v_{C2}(t) \)  
(2)

KCL: \( i_L(t) = i_{R1}(t) + i_{C1}(t) + i_{C2}(t) \)  
(3)

Now, use \( v_{R2} = R_2 i_{C2} \) and \( i_{C2} = C_2 \frac{dv_{C2}}{dt} \) in (2), so that \( v_{C1}(t) \) is expressed only in terms of \( v_{C2}(t) \) and its derivative. Then use \( i_{R1}(t) = \frac{v_{C1}}{R_1} \) and \( i_{C1}(t) = C_1 \frac{dv_{C1}}{dt} \) in (3). Finally, use \( v_L(t)L \frac{di_L}{dt} \) in (1), yielding a 3rd-order differential equation in \( v_{C2}(t) = v_0(t) \).

\[
\frac{d^3 v_{C2}}{dt^3} + \left( \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_1 R_2} \right) \frac{d^2 v_{C4}}{dt^2} + \left( \frac{1}{LC_1} + \frac{1}{C_1 C_2 R_1 R_2} \right) \frac{dv_{C4}}{dt} + \frac{1}{R_2 L C_1 C_2} v_{C4} = \frac{1}{R_2 L C_1 C_2} v_i.
\]

3. Let the circuit elements have parameter values \( L = 0.025 \) H, \( C_1 = 0.25 \) \( \mu \)F + \( \alpha \times 1 \) \( \mu \)F, \( R_1 = 1,000 \) ohms, \( R_2 = 1,000 \) ohms, \( C_2 = 0.05 \) \( \mu \)F + \( \alpha \times 0.2 \) \( \mu \)F, where \( \alpha = 0.0xyz \), with \( xyz \) the final three digits of your student ID number. The value of \( 0.0xyz \) satisfies \( 0 \leq 0.0xyz \leq 1 \), so the \( C_1 \) capacitance value lies in the range \([0.25, 1.25]\) \( \mu \)F, and the \( C_2 \) capacitance value lies in the range \([0.05, 0.25]\) \( \mu \)F. Use Matlab to determine the response of the circuit \( v_0(t) \) over a suitable time interval (roughly \([0, 0.010]\) sec) to input \( v_i(t) = u(t) \) V, where \( u(t) \) is the unit-step signal. Accurately plot the response, \( v_0(t) \), and determine (from the numerical response) the 100% rise time, percent overshoot, and 2% settling time. (Note that the default rise time is the 10% to 90% rise time set by some Matlab functions. Make sure your numerical result matches your plot and corresponds to the 100% rise time.) Also, accurately plot \( v_L(t) \). Verify that \( v_0(t) \) and \( v_L(t) \) approach the correct steady-state values as \( t \to \infty \).

Plot of unit step response for \( R_1 = R_2 = 1,000 \) \( \Omega \), \( L = 0.025 \) H, \( C_1 = 0.25 \) \( \mu \)F, \( C_2 = 0.05 \) \( \mu \)F, and \( C_1 = 1.25 \) \( \mu \)F, \( C_2 = 0.25 \) \( \mu \)F. The respective rise time, settling time, and percent overshoot are labeled in the figures, and range as \( t_r \in [0.52, 0.198] \) ms, \( M_p \in [37, 49]\% \), and \( t_s \in [1.7, 5.0] \) ms.
function projS2019Part1(R1,R2,L,Cap1,Cap2)
% 3rd-order passive filter with parallel RC
% combination at center node.
% projS2019(1000,1000,0.025,0.25e-6,0.05e-6)
% b0=1/(R2*L*Cap1*Cap2);
b=[b0];
a2=1/(R1*Cap1) + 1/(R2*Cap2) +1/(R2*Cap1);
a1=1/(R1*R2*Cap1*Cap2) + 1/(L*Cap1);
a0=1/(R2*L*Cap1*Cap2);
% tranfer function method v_0(t) only
a=[1 a2 a1 a0];
sys=tf(b,a);
t=[0:1e-7:10e-3];
vi=ones(1,length(t));
[y,x]=lsim(sys,vi,t);
figure(3)
plot(t,y(:,1))
title('Unit Step Response (From Transfer Function)')
xlabel('Time, t, sec')
ylabel('v_0(t), Volts')
% state variables approach, v_0(t) and v_L(t)
A=[0 1 0;0 0 1;-a0 -a1 -a2];B=[0;0;b0];
C=[1 0 0; -1 -R2*Cap2 0];D=[0;1];
[y1,x1]=lsim(A,B,C,D,vi,t,[0;0;0]);
figure(1)
subplot(2,1,1)
plot(t,y1(:,1))
title('Unit Step Response, v_0(t)')
xlabel('Time, t, sec.')
subplot(2,1,2)
plot(t,y1(:,2))
title('v_L(t)')
xlabel('Time, t, sec.')
ylabel('Voltage, v_L(t)')