This course project is intended to be an individual effort project. The student is required to **complete the work individually, without help from anyone else.** (The student may, however, get help from the instructor or TA.) The student must turn in this page with the report, including signature on the pledge at the bottom. **The final project report is due at the beginning of class lecture on Wednesday, April 17.** Late reports may be turned in on Friday, April 19, for a 20% reduction in grade. The project tasks are outlined on the following pages. The student must produce a typed report, including equations, figures and tables, that includes discussion of the results and any conclusions reached. The report may contain a comparison of techniques, a discussion of the behavior of the circuit as a filter, and any positive outcomes of the efforts of the project. The guidelines for good technical report writing need to be followed. We are particularly interested in reports demonstrating abilities related to the following ABET Criteria 3 outcomes: (a) an ability to apply knowledge of mathematics, science, and engineering; (b) an ability to design and conduct experiments as well as analyze and interpret data; (g) an ability to communicate effectively; (k) an ability to use techniques, skills, and modern engineering tools necessary for engineering practice.

**Notices (standard School EE 321 project requirements):**
1) A preliminary report, consisting of Part I, tasks 1), 2) and 3), is due Wednesday, March 27, at the beginning of class lecture. For this preliminary due date, turn in only the results, the MATLAB commands, and a signed copy of the pledge below; no typed report is needed. Your results should present a derivation of the differential equation and the state space model. This preliminary report is worth 10 points out of the total 100 points on the project grade. You may turn in Part I on Friday, March 29 for a 20% reduction in that part of the grade (i.e., for a maximum of 8 points). The final report must also include all of Part I.
2) Any evidence of collaboration with other students on either the final project report or the preliminary results will result in a grade of zero for the project for all collaborators. Additionally, the violation will be reported to the School of EECS and the University.
3) Write the report in your own words; do not use sentences or paragraphs from any other source. Grading will be based, in part, on content and grammar, so be sure to proof read your report. If significant portions of the report are found to be copied from another source, without proper attribution, a grade of zero will be assigned for the project and the plagiarism reported to the School and University.
4) The pledge below must be signed and turned in with both the preliminary results and the final project report. Final project reports without the signed pledge will receive a grade of zero.

**PLEDGE:** I HAVE NOT OBTAINED ASSISTANCE IN COMPLETING THIS PROJECT FROM ANYONE OTHER THAN THE INSTRUCTOR OR TA FOR THIS COURSE, NOR HAVE I GIVEN ASSISTANCE TO OTHER MEMBERS OF THIS CLASS.

**SIGNED:** ___________________________________________________________
A circuit with input voltage $v_i(t)$, and output voltage $v_o(t)$, is shown below, with complex impedances in the circuit representing either a resistor ($Z(s) = R$), an inductor ($Z(s) = sL$), or a capacitor ($Z(s) = 1/sC$), or some combination of any of the three.

![Generic circuit](image)

Figure 1. Generic circuit with input voltage $v_i(t)$, and output voltage $v_o(t)$.

**Part 1.**

1. Use $s$-domain techniques and determine the transfer function
   $$H(s) = \frac{V_o(s)}{V_i(s)}$$
   in terms of the impedances $Z_1, Z_2, Z_3,$ and $Z_4$.

2. Let $Z_1(s) = sL$, $Z_2(s) = R_1||\frac{1}{sC_1}$, $Z_3(s) = R_2$, and $Z_4(s) = 1/sC_2$, corresponding to the circuit below. Using Kirchhoff’s voltage and currents laws, derive the $3^{rd}$ order differential equation for $v_o(t)$. Assume that there is no energy stored in the capacitors or inductor at time $t = 0^-$. From the differential equation, find a state variable representation, and specify the state variable matrices A, B, C, D, to generate the voltages $v_o(t)$ and $v_L(t)$. From the differential equation, determine the transfer function, $H(s) = \frac{V_o(s)}{V_i(s)}$. Verify that it is equivalent to the result in 1) for the choice of impedances in 2).

3. Let the circuit elements have parameter values $L = 0.025 \, \text{H}, C_1 = 0.25 \, \mu F + \alpha \times 1 \, \mu F$, $R_1 = 1,000 \, \text{ohms}$, $R_2 = 1,000 \, \text{ohms}$, and $C_2 = 0.05 \, \mu F + \alpha \times 0.2 \, \mu F$, where $\alpha = 0.\text{xyz}$, with $\text{xyz}$ the final three digits of your student ID number. The value of $0.\text{xyz}$ satisfies $0 \leq 0.\text{xyz} \leq 1$, so the $C_1$ capacitance value lies in the range $[0.25, 1.25] \, \mu F$, and the $C_2$ capacitance value lies in the range $[0.05, 0.25] \, \mu F$. Use Matlab to determine the response of the circuit ($v_o(t)$) over a suitable time interval (roughly $[0, 0.010] \, \text{sec}$) to input $v_i(t) = u(t) \, V$, where $u(t)$ is the unit-step signal. Accurately plot the response, $v_o(t)$, and determine (from the numerical response) the 100% rise time, percent overshoot, and 2% settling time. (Note that the default rise time value that some Matlab functions provide is
the 10% to 90% rise time, so make sure your numerical result matches your plot and corresponds to the 100% rise time.) Also, accurately plot \(v_L(t)\) and \(v_0(t)\) approach the correct steady-state values as \(t \to \infty\).

**Part II.**

1. Using the circuit and transfer function from task 2), find all finite poles and zeros and plot their location in the s-plane. Also, accurately plot the transfer function frequency response (use Bode plots). Verify that the steady-state output level of the unit step-response matches the “dc gain” of the frequency response. Use Matlab to find the response to input \(v_i(t) = 20 \cos(10,000 \ t) u(t)\). Evaluate the response for a suitable length of time so that the response reaches steady state. Then, determine the magnitude and phase of the output, \(v_o(t)\), and compare to the magnitude and phase of your Bode plots at frequency 10,000 rad/sec. Include in your figure a plot of the theoretical expression for the sinusoidal steady-state response. (Note: The Matlab sine and cosine functions require that a phase angle be expressed in radians.)

2. Set \(R_1 = \infty\) (so that resistor \(R_1\) drops out of the circuit) and let the value of resistor \(R_2\) be \(R_2 = 1,000 + 9,000 \times 0.\text{xyz}\) ohms, where again \(\text{xyz}\) is the last three digits of your student ID number. (The resistance then lies in the range [1,000 to 10,000] ohms.) Design the filter to be a 3rd-order Butterworth filter with \(-3\) dB cutoff frequency 12 kHz (24,000\(\pi\) radians/sec). This will involve, for your specified value of \(R_2\), properly selecting the values of \(C_1\), \(C_2\), and \(L\). Note that the desired Butterworth filter transfer function must have the form \(H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c^2 s^2 + 2\omega_c s + \omega_c^3}\), where \(\omega_c\) is the cutoff frequency (24,000\(\pi\) radians/sec). After designing the filter, complete the following.

   a. Find the pole locations (e.g., using MATLAB), and plot the pole locations in the s-plane. Identify the angle of the complex poles in the s-plane and verify that they correspond to a 3rd-order Butterworth filter.

   b. Use state variables and MATLAB to determine the unit step response. Accurately plot the step response and determine the 100% rise time, maximum percent overshoot, and 2% settling time.

   c. Accurately plot the filter frequency response (use Bode plots). Verify that the filter response matches the desired Butterworth filter frequency response.

   d. Without computer assistance (that is, by hand calculation), determine the equation for the impulse response, \(h(t)\), the output voltage in response to a Dirac impulse input, \(v_i(t) = \delta(t)\).

   e. Without computer assistance, determine the equation for the unit-step response. Verify that your equation for unit-step response matches the MATLAB state variable solution in task 5b).

   f. The steady-state sinusoidal response of the filter is determined by the frequency response, evaluated at the sinusoid frequency. Let the circuit input be \(v_i(t) = \sin(2\pi 12000 \ t) u(t)\). Use Matlab to compute the filter response for a suitably long duration so that the response reaches (approximately) the steady-state signal. Plot in a single figure both \(v_i(t)\) and \(v_o(t)\), and identify in your figure that the expected filter magnitude and phase are realized. One way to show this would be to plot the theoretical sinusoidal steady state solution (also in the
same figure) and show that $v_0(t)$ approaches this as time gets sufficiently large. Verify that the results match your Bode plots. Approximately how long does it take for the (Matlab) response to reach steady-state? Compare this (observed) time to the settling time found in part 5b).

Part III.

3. A 3rd-order Butterworth high-pass filter has transfer function

$$H_{highpass}(s) = \frac{s^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3},$$

where $\omega_c$ is the high-pass filter cutoff (-3 dB) frequency.

a. Design an active circuit (that is, using operational amplifiers) 3rd-order Butterworth high-pass filter with cutoff frequency 2,000 Hz, and a variable gain between 0 and 40 dB. Use as your design a cascade of one first-order section, one second-order section, and one all-pass section to adjust the overall gain. Use the capacitor value $C = 1 + 9 \times \text{xyz}$ nF for your design, where, again, $\text{xyz}$ is the last three digits of your student ID number. Provide the relevant details of the design, including the values of any magnitude or frequency scale factors used, a summary of the selected component values, and a diagram of the final op amp circuit with labeled component values.

b. Assume that the selected resistor and capacitor values are subject to 10% tolerance (that is, each may, independently, vary by up to ±10%). Analyze your design, determine how pole locations are affected by changes in the component values, and determine approximate worst case variation in the filter cutoff frequency. Provide Bode plots for the extreme cases. How much can the cutoff frequency vary? Can the general magnitude frequency response shape vary significantly from the 3rd-order Butterworth characteristic?

Report Preparation.

In your report, present the results of the above tasks along with the discussion mentioned on the first page, and supporting derivations, analysis, design equations, plots, and MATLAB code. As a rule-of-thumb, the report should be sufficiently detailed so that if you were to refer to it in one year, you could easily follow the derivations, discussion, and results. Since the report must be typeset, select a suitable word processing system for your work. The LaTeX system is available for free download and is excellent, but requires some learning.

Note: It is common practice for students to use this project report as part of their written work submitted to satisfy the University’s Writing Portfolio requirement. So, keep in mind that the report might, later, be evaluated for that purpose.