1. (20 points) Design a first-order, RL high-pass filter to have (-3 dB) cutoff frequency 20,000 rad/sec. Use a 40 mH inductor. Specifically,
   a. (5 points) Determine the transfer function, $H(s)$.
   b. (5 points) Determine the required value of resistor, $R$.
   c. (5 points) Sketch the magnitude Bode frequency response plot, accurately labeling the cutoff frequency.
   d. (5 points) Determine the steady-state response to input $v_i(t) = 2 \ u(t)$ volts.
2. (20 points) Find \( y(t) = x(t) \ast h(t) \), the convolution of the signals \( x(t) \) and \( h(t) \), where
\[
x(t) = 2 \ [u(t) - u(t - 6)] = \begin{cases} 
2, & 0 \leq t < 6; \\
0, & \text{otherwise};
\end{cases}
\]
and
\[
h(t) = 4 \ [u(t) - u(t - 3)] = \begin{cases} 
4, & 0 \leq t < 3; \\
0, & \text{otherwise};
\end{cases}
\]
Sketch \( y(t) \), accurately labeling relevant amplitudes and x-axis values.

3. (10 points) A certain circuit has transfer function \( H(s) = \frac{V_o(s)}{V_i(s)} = \frac{100s}{s^2 + 100s + 10,000} \). Find the steady-state response to the input \( v_i(t) = 240 \cos(100 \ t + 15^\circ) \).
4. (20 points) A circuit with input voltage \( v_i(t) \) and output voltage \( v_o(t) \) is shown below. Assume that zero energy is stored in the circuit at time \( t = 0^- \), and that \( C = 1 \ \mu F, \ R = 5000 \ \Omega \). The circuit has transfer function \( H(s) = \frac{V_o(s)}{V_i(s)} \).

a. (10 points) Find the impulse response, \( h(t) \).

b. (10 points) Find the response to input \( v_i(t) = 16 \ u(t) \) V.

\[ v_i(t) \quad + \quad C \quad \quad R \quad \quad v_o(t) \quad + \quad \quad - \]

\[ v_i(t) \quad + \quad C \quad \quad R \quad \quad v_o(t) \quad + \quad \quad - \]
5. (30 points) A second-order filter is shown below. Assume that there is no energy initially stored in the circuit.

\[ \frac{1}{s^2 + \frac{2\xi\omega_c}{C}s + \omega_c^2} \]

a. (5 points) Find \( I(s) \) in terms of \( V_i(s) \) and the circuit parameters.

b. (5 points) Use the final value theorem to find \( \lim_{t \to \infty} i(t) \) for \( v_i(t) = 12 \ u(t) \ V \).

c. (5 points) Determine the filter transfer function, \( H(s) = \frac{V_o(s)}{V_i(s)} \).

d. A second-order low-pass filter with complex-conjugate poles has transfer function of the form \( H(s) = \frac{\omega_c^2}{s^2 + 2\xi\omega_c s + \omega_c^2} \). Fill in the blanks:

i. (5 points) For the filter to be a Butterworth filter, the poles must lie at angles \( \pm \_____ \) degrees with respect to the negative real axis.

ii. (5 points) For \( H(s) \) to be a Butterworth filter, the value of the damping constant must be \( \xi = \______ \).

e. (5 points) Design the filter to be a second-order Butterworth filter with cutoff (-3 dB) frequency 2,000 rad/sec. Use a 1 \( \mu F \) capacitor and specify the required values of \( R \) and \( L \).
Useful Formulae

\[
\begin{align*}
v &= L \frac{di}{dt} \quad i = C \frac{dv}{dt} \quad v = iR \quad F = ma \quad c = 2\pi \ r \quad E = mc^2 \\
L\{u(t)\} &= \frac{1}{s} \quad L\{tu(t)\} = \frac{1}{s^2} \quad L\{e^{-at}u(t)\} = \frac{1}{s + a} \\
L\{\sin(\omega t)\} u(t)\} &= \frac{\omega}{s^2 + \omega^2} \quad L\{\cos(\omega t)\} u(t)\} &= \frac{s}{s^2 + \omega^2} \\
L\{e^{-at}\sin(\omega t)\} u(t)\} &= \frac{\omega}{(s + a)^2 + \omega^2} \quad L\{e^{-at}\cos(\omega t)\} u(t)\} &= \frac{s + a}{(s + a)^2 + \omega^2} \\
L\left(\frac{dx(t)}{dt}\right) &= sX(s) - x(0^-) \\
L\left(\frac{d^2x(t)}{dt^2}\right) &= s^2X(s) - sx(0^-) - \frac{dx(0^-)}{dt} \\
\lim_{t \to \infty} x(t) &= \lim_{s \to 0} sX(s) \\
\lim_{t \to 0^+} x(t) &= \lim_{s \to \infty} sX(s) \\
\int_0^x x(t) dt &= \frac{X(s)}{s} \\
e^{-at}x(t) &\to X(s + a) \\
x(at), \ a > 0 &\to \frac{1}{a} X\left(\frac{s}{a}\right) \\
tx(t) &\to -\frac{dX(s)}{ds} \\
x(t-a)u(t-a), \ a > 0 &\to e^{-\alpha t}X(s) \\
\frac{\beta s}{s^2 + \beta s + \omega_0^2} &\to \omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}, \quad \omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} \\
20 \log_{10} |H(j\omega)| &\quad \theta = \tan^{-1} \frac{\text{Im}\{H(j\omega)\}}{\text{Re}\{H(j\omega)\}} \quad |H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}
\end{align*}
\]