Let \( x_a(t) = e^{-100t}u(t) \), and generate the samples \( x(n) = x_a(nT) = e^{-100nT} \), \( n = 0,1,\ldots,N-1, N = 100 \). Compare the Fourier transform of \( x_a(t) \), namely, \( X_a(\omega) = \frac{1}{100+j\omega} \) to the fast Fourier transform (FFT) computed on the \( N \) samples. If there is negligible aliasing in the sampling, and negligible truncation of the energy of \( x_a(t) \) for the interval sampled, then the FFT gives samples (approximately) of

\[
\frac{1}{T} \sum_{m=-\infty}^{\infty} X_a \left( \omega - \frac{2\pi m}{T} \right) \sim \frac{1}{T} \left[ X_a(\omega) + X_a(\omega - \frac{2\pi}{T}) \right]
\]

at the frequencies \( \omega = \frac{2\pi k}{NT} \), \( k = 0,1,\ldots,N-1 \).

Using the Matlab code appended, for \( T = 0.001 \), the plots of \( X_a(\omega)/T \) and the FFT vs \( \omega \) are shown below. Note that the FFT results are very similar to the values of \( X_a(\omega)/T \) for frequencies out to \( \frac{\pi}{T} \) rad/s.
Reducing the sampling rate to $\frac{1}{T} = 100$ Hz yields the plot below. Note the effect of the aliasing (since the sampling rate is too small, relative to the effective bandwidth of the signal).

```
function [ output_args ] = samp_spect(T)
% This generates the fft and plot of the spectrum
% of x(t) = exp(-100t)u(t). Sampling rate 1/T
n=[0:99];  % 100 samples
x=exp(-100*n*T);  % samples of x(t) = exp(-100t)u(t)
N=length(n);
X=fft(x);
w=2*pi*[0:N-1]/(N*T);  % FFT frequency samples
Xa=1./(100+j*w);  % Fourier transform of x(t)
figure(1)
plot(w,abs(X),w,abs(Xa)/T,'-.')  % Note scaling by 1/T
title('FFT of x_a(t) sampled at rate 1/T = 100, and X_a(\omega)')
xlabel('Frequency, \omega, rad/s')
ylabel('|FFT| and |X_a(\omega)|')
end
```