1. A discrete information source is modeled as a discrete random variable, $X$, with sample space $S = \{0,1,2,3,4,5\}$ and probabilities $P(x) = \{0.6,0.16,0.08,0.07,0.05,0.03\}$ for $x = \{0,1,2,3,4,5\}$.
   a. Determine the entropy, $H(X)$.
   b. Design a Huffman code for the source. List the codewords in your code. Determine the average codeword length, and compare to the entropy.
   c. Let a source sequence be (2,4,0,3,5,0,1) and determine the corresponding encoded bitstream.
   d. Suppose that the 3rd bit in the bitstream in part c) is received in error, and determine the resulting sequence of decoded symbols.

2. A facsimile source is modeled as a memoryless binary random variable, $X \in \{0, 1\}$, with $P(0) = 0.95$.
   a. Determine the entropy, $H(X)$.
   b. Design Huffman codes for encoding i) three source letters together and ii) four source letters together. Determine the average codeword length per source letter for each case, and compare to the entropy.

3. Let $X$ and $Z$ be independent Gaussian random variables, with respective means $\mu_X, \mu_Z$ and variances $\sigma_X^2, \sigma_Z^2$.
   a. Let $Y = X + Z$. Find the probability density function for $Y$.
   b. Let $\mu_Z = 0$ and let $V = -1 + Z$. Determine the pdf for $V$. Use this to determine the probability that $V > 0$. Express in terms of the Q-function.

4. A Shannon code is a prefix-free lossless code with the property that if source letter $x$ has probability $P(x)$, then the corresponding codeword has length $\lceil -\log_2 P(x) \rceil$ bits. For the source in problem 1, design a Shannon code for encoding the source. List the codewords for each source symbol and determine the average codeword length for your code. Compare the average codeword length to the source entropy and to the average codewords length for the Huffman code constructed in problem 1.

5. A certain board game is played using two fair six-sided die. The outcome of each throw of either die is modeled as an independent random variable with 6 possible values, the numbers 1,2,3,4,5, or 6, each of probability 1/6. The probability mass function, $p_{X_i}$, for the toss of a die is then $P(x) = \begin{cases} \frac{1}{6}, & x \in \{1,2,3,4,5,6\}, \\ 0, & \text{otherwise} \end{cases}$, that is, uniform on the sample space. Let $X_1, X_2$ denote the toss of the two die, and let $Y = X_1 + X_2$ be the sum of the outcomes. Find the probability mass function for $Y$. Verify that this is $p_{X_1} \ast p_{X_2}$.

6. A different board game is played with one 4-sided die, one 8-sided die, and one 12-sided die. Assume that each die is “fair” so that the probability of each side is one divided by the number of sides for the die. Let $X_1, X_2, X_3$ be independent random variables, modeling the outcome of a throw of each respective die. For example, $X_1$ is uniformly distributed on the sample space $\{1,2,3,4\}$, $X_2$ is uniformly distributed on $\{1,2,\ldots,8\}$, etc.
   a. Let $Y = X_1 + X_2$ and find the probability mass function for $Y$.
   b. Let $Z = X_1 + X_2 + X_3$, and find the probability mass function for $Z$. 