1. A linear, time-invariant system with input, \( x(t) \), and output, \( y(t) \), has the unit-step response, \( c(t) \), shown below.

a. Find, and accurately sketch, the system impulse response, \( h(t) \).

b. Find, and accurately sketch, \( y(t) = h(t) \ast [u(t) - u(t-2)] \).

2. A discrete-time, linear, time-invariant system with input \( x(n) \) and output \( y(n) \) has the impulse response \( h(n) = [2, 2, 1, -1] \), where the underline denotes the \( n = 0 \) sample, and all signal values outside the range specified are zero.

a. (2 points) Is the system causal? (You must provide correct justification to receive credit.)

b. (3 points) Is the system BIBO stable? (You must provide correct justification to receive credit.)

c. The system input is \( x(n) = [-1, -1, 0, 1, 1] \), where the underline denotes the \( n = 0 \) sample, and all signal values outside the range specified are zero.

i. (2 points) \( y(n) \) begins at \( n = ? \)

ii. (3 points) \( y(n) \) ends at \( n = ? \)

iii. (10 points) Find \( y(n) \) for all \( n \).
3. A discrete-time system with input, $x(n)$, and output, $y(n)$, is described by $y(n) = 2x(n) - 1$.  
   a. (5 points) Find the system response to the (Kronecker) impulse input, $\delta(n)$.  
   b. (5 points) Determine if the system is BIBO stable. Proper justification must be provided to receive credit.

4. A continuous-time system with input, $x(t)$, and output, $y(t)$, has the input-output description $y(t) = 2 \cos(t) |x(t)|$. Provide yes/no answers to the following by circling the correct answer. Determine if the system is  
   a. Memoryless?  
   b. Causal?  
   c. BIBO stable?  
   d. Linear?  
   e. Time-invariant?
A linear, time-invariant system with input, \( x(t) \), and output, \( y(t) \), has the impulse response, \( h(t) \), and the unit-step response, \( c(t) \), shown below.

Find, and accurately sketch, the response to the input \( x(t) = 2\delta(t - 1) - u(t - 3) \).
Consider the composite system below, constructed from distinct linear, time-invariant systems.

a. (10 points) Determine the overall impulse response, \( g(t) \), in terms of \( h(t) \) and \( r(t) \).

b. (10 points) Let \( h(t) = u(t) - u(t-1) \) and \( r(t) = \delta(t-1) \). Find \( y(t) \) if the system input is \( x(t) = u(t) \).

A discrete-time, linear, time-invariant system with input, \( x(n) \), and output, \( y(n) \), has impulse response \( h(n) \).

a. If \( h(n) = (1, 2, -1) \), use convolution to find \( y(n) = h(n) * x(n) \), for \( x(n) = [u(n) - u(n-6)] \).

b. If

\[
h(n) = \left(\frac{1}{2}\right)^n u(n),
\]

use convolution to find \( y(n) = h(n) * x(n) \), for \( x(n) = [u(n) - u(n-6)] \).
A periodic signal, \( x(t) \), has period \( T = \frac{\pi}{5} \) sec, fundamental frequency \( \omega_0 = \frac{2\pi}{T} = 10 \) rad/s, and exponential Fourier series coefficients

\[
c_k = \begin{cases} 
5, & k = 0 \\
10, & k = 1, -1 \\
-7j, & k = 2 \\
7j, & k = 2 \\
-3, & k = 3, -3 \\
0, & \text{otherwise}
\end{cases}
\]

d. Find the power in the signal \( x(t) \).
e. A truncated Fourier series is of the form

\[
x_N(t) = \sum_{k=-N}^{N} c_k e^{j2\pi k t / T}.
\]

Find the normalized mean-square truncation error for

i. \( N = 1 \).

ii. \( N = 5 \).
f. Accurately sketch the magnitude spectrum for \( x(t) \), labeling all relevant amplitudes.
g. The signal, \( x(t) \), is applied as input to a linear, time-invariant system with output, \( y(t) \). The system has impulse response \( h(t) \) with Fourier transform (the system frequency response) \( H(\omega) \), where

\[
H(\omega) = \begin{cases} 
20, & 5 \leq |\omega| \leq 25 \\
0, & \text{otherwise}
\end{cases}
\]

Find the Fourier series for \( y(t) \). Express \( y(t) \) in simplest form (e.g., sines and cosines).
A linear, time-invariant (LTI) system with overall impulse response \( h(t) \) is constructed from three simpler LTI subsystems, as shown below, with respective impulse responses \( g_1(t) \), \( g_2(t) \), and \( g_3(t) \).

a. Find \( h(t) \) in terms of \( g_1(t) \), \( g_2(t) \), and \( g_3(t) \).

b. If \( g_1(t) = \delta(t - 2) \), \( g_2(t) = 6\delta(t - 3) \), and \( g_3(t) = -2\delta(t - 1) \), find \( h(t) \).

c. Suppose that \( h(t) = \delta(t + 1) + 2\delta(t - 3) \). Find, and accurately sketch \( y(t) = x(t) \ast h(t) \) for \( x(t) = [u(t + 2) - u(t - 2)] \).

d. Suppose that \( h(t) = 2e^{-2t}u(t) \). Find, and sketch, \( y(t) = x(t) \ast h(t) \) for the same \( x(t) \) as in part c), \( x(t) = [u(t + 2) - u(t - 2)] \).
1. A periodic signal, $x(t)$, is shown below.

![Periodic Signal Diagram](image)

i. Find the exponential Fourier series, $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T}$
   i. Period $T =$
   ii. $c_0 =$
   iii. $c_k =$

ii. Let $z(t) = 3x(t) + 6$. Find the exponential Fourier series for $z(t)$ in terms of the Fourier series coefficients for $x(t)$.

iii. The signal $x(t)$ in part a) is applied as the input to a linear, time-invariant system with output $y(t)$ and transfer function $H(s) = \frac{0.2}{s+0.4\pi}$.
   i. Find the Fourier series for $y(t)$.
   ii. Find the ratio of the amplitude of the 1st harmonic of the Fourier series for $y(t)$ to the amplitude of the 1st harmonic of the Fourier series for $x(t)$.
The square wave signal, \( x(t) \), shown below, has period \( T \) sec and exponential Fourier series

\[
x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T},
\]

where the Fourier series coefficients are given by

\[
c_k = \begin{cases} 
1, & k = 0 \\
sinc \left( \frac{k}{2} \right), & k \neq 0
\end{cases}
\]

with \( \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \), and the fundamental frequency \( \omega_0 = \frac{2\pi}{T} \) rad/s.

iv. Find the Fourier transform of \( x(t) \), and plot the magnitude spectrum, \( |X(\omega)| \), carefully labeling important amplitudes and frequency axis intercepts.

v. Let \( T = 0.0625 \) sec (so \( \omega_0 = \frac{2\pi}{T} = 32\pi \) rad/s). The signal \( x(t) \) is applied as input to a linear, time-invariant system with output \( y(t) \). The system frequency response is shown below. Find an explicit expression for \( y(t) \).
9. Analog signal \( x(t) = 2 \cos(32\pi t) - 4 \sin(7\pi t) \).

vi. Find \( X(\omega) \) and accurately sketch \( X(\omega) \), carefully labeling important amplitudes and frequency axis intercepts.

vii. The signal \( x(t) \) is sampled at rate \( F_s = \frac{1}{T} \) samples/sec, and the analog signal \( y(t) \) is formed as shown in the block diagram below, \( y(t) = \sum_{n=-\infty}^{\infty} x(nT)h(t-nT) \), where \( h(t) \rightarrow H(\omega) \) and \( H(\omega) = T \frac{P_{\pi}(\omega)}{T} = \begin{cases} T, & |\omega| \leq \frac{\pi}{T} \\ 0, & \text{otherwise} \end{cases} \)

\[
\begin{array}{c}
\text{Sampler} \\
\text{Rate} = \frac{1}{T} \\
x(nT)
\end{array} \quad \begin{array}{c}
x(nT) \\
\text{Reconstruct} \\
H(\omega)
\end{array} \quad \begin{array}{c}
y(t)
\end{array}
\]

i. From the sampling theorem, what is the smallest rate at which \( x(t) \) can be sampled and the reconstructed signal is \( y(t) = x(t) \).

ii. Let \( F_s = \frac{1}{T} = 25 \) samples/sec. Sketch the spectrum at the output of the sampler. Find an explicit (time-domain) expression for \( y(t) \).

10.

viii. Use Fourier transform pairs and properties to find the Fourier transform of the signal \( x(t) = 10e^{-3|t|} \cos(30\pi t) \), where \( u(t) \) is the unit step function.

ix. Sketch \( |X(\omega)| \), carefully indicating important amplitudes and frequencies.
A discrete source is modeled as a discrete random variable, $X$, with 6 outcomes in the sample space $S = \{0,1,2,3,4,5\}$.

a. If $P(x) = \frac{1}{6}$, for $x = 0,...,5$, determine the entropy, $H(X)$. (Note that, $\log_2 2 = 1.0$, $\log_2 3 = 1.5850$, and $\log_2 6 = 2.5850$.)

b. Let the symbol probabilities be $P(x) = (0.35, 0.25, 0.12, 0.12, 0.08, 0.08)$ for $x = 0,...,5$, with the source entropy, $H(X) = -\sum_{x=0}^{6} P(x) \log_2 (P(x)) = 2.3473$ bits. Design a Huffman code for the source. List all codewords, determine the average codeword length, and compare the average codeword length to the entropy.

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<th>Codeword</th>
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<th>P(x)</th>
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<tr>
<td>5</td>
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