4 types of signal

a) continuous-time
   i) continuous-amplitude          ii) discrete-amplitude

b) discrete-time
   iii) continuous-amplitude      iv) discrete-amplitude
A few special signals

(Kronecker) impulse signal $\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

Unit-step signal $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

Note: Analogous to

$u(n) = \sum_{k=-\infty}^{n} \delta(k)$

$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$

$\delta(n) = u(n) - u(n-1)$

$\delta(t) = \frac{du(t)}{dt}$

Exponential signal

$x(n) = a^n u(n)$
Operations on signals

Addition of signals means term-by-term addition:
\[ x(n) + y(n) \Rightarrow (\ldots, x(-1) + y(-1), x(0) + y(0), \ldots) \]

Multiplication of signals means term-by-term multiplication:
\[ x(n)y(n) \Rightarrow (\ldots, x(-1)y(-1), x(0)y(0), \ldots) \]

Delay by \( n_0 \) samples: \( x(n - n_0) \)

Important sum
\[
\sum_{n=0}^{N-1} a^n = \begin{cases} 
1 - a^N, & a \neq 1 \\
\frac{1}{N}, & a = 1
\end{cases}
\]
System Properties

Memoryless (static) –

Linear –

Time-invariant (shift-invariant) –

Causal –

BIBO Stable –
Ex 1. $y(n) = x(n) + 1$. Is this system memoryless, causal, time-invariant, linear, BIBO stable?

Ex. 2. $y(n) = T[x(n)] = 2nx(n)$. Is this system M,C,TI,L,BIBO stable?

Ex. 3. $y(n) = T[x(n)] = x^2(n) - x^2(n - 1)$. Is this system M,C,TI,L,BIBO stable?
Ex. 4. \( y(n) = \text{median}\{x(n - 1), x(n), x(n + 1)\}. \) Is this system M,C,TI,L,BIBOstable?

Find \( y(n) = \text{median}\{x(n - 1), x(n), x(n + 1)\} \) for \( x(n) = \{\cdots, 0, 0, 3, 2, 1, -3, 2, 1, 4, 0, 0, \cdots\} \) with \( x(0) = -3. \)

What are possible applications?
Below is a test image called “Goldhill” (Gold Hill, Shaftesbury, Dorset, England)

(512x512 8-bit “grayscale” or luminance image)
Adding “salt and pepper” noise yields the image below.

There are various approaches to filtering to reduce the visual effect of the “salt and pepper” noise.

Approach 1: Use a lowpass filter (e.g., 3x3 impulse response) – a linear filtering solution.

Approach 2: Use a median filter (e.g., a 3x3 mask) – a nonlinear filtering solution.
Below is the result of simple lowpass filtering (3x3 impulse response)
Compare linear lowpass filtering to median filtering. Below is the result of simple lowpass filtering (3x3 impulse response)

Below is the result of simple median filtering (3x3 mask)