1. Let $x_a(t) = 400e^{-800|t|}\sin(2\pi f_0t)$, where $f_0 = 104$ kH. The signal $X_a(f)$ is a band-pass signal.

(a) Let $v(t) = 400e^{-800|t|}$ and find/plot $V(f)$. From the plot, determine the approximate signal bandwidth based on the frequency at which $|V(f)|$ has value 30 dB less than the peak magnitude. (This corresponds to measuring bandwidth at frequencies where the magnitude is roughly 3% of the peak magnitude. For the remainder of the problem, interpret bandwidth as being the range of (positive) frequencies outside of which the magnitude frequency response is less than 30 dB of the peak magnitude. That is, measure bandwidth at the level of -30 dB relative to the peak magnitude.)

(b) Find $X_a(f)$. Determine the approximate bandwidth, using the frequencies where the magnitude of $X_a(f)$ is down 30 dB from the peak. From standard Nyquist sampling theory, what is the Nyquist sampling rate?

(c) Let $f_1 = 100$ kHz and $f_2 = 110$ kHz. Assume that the signal is first bandpass filtered to the frequency range $\{f_1, f_2\}$, and select a suitable bandpass sampling rate.

(d) Use Matlab to accurately plot $|X_a(f)|$.

(e) Use Matlab to simulate the bandpass sampling. Specifically, let the simulation sampling rate be 1 MHz, use the bandpass sampling theory to select a suitable bandpass sampling rate $1/T_{BP}$, and sample $x_a(t)$ over the range -10 ms to 10 ms. Plot the sampled waveform. Note that in doing so, no explicit bandpass filtering is done prior to bandpass sampling (and, none is necessary because the signal construction is limited to only $X_a(f)$.) What translate appears in the baseband range, $[0, 1/(2T_{BP})]$?

(f) In typical applications, out-of-band signal content is typically present, so that bandpass filtering, prior to bandpass sampling, is desirable. To examine the importance of the bandpass filtering, assume that the sampled bandpass signal is sampled in the presence of independent and identically distributed Gaussian noise. So, modify each sample of the 1 MHz sampled bandpass signal in part e) to form (in Matlab)

$$x(n) = x_a(nT) + sig \ast \text{randn}(n);$$

where $1/T = 10^6$ samples/sec, sig = 20 is the Gaussian noise sample standard deviation, and randn() is the Matlab Gaussian random noise generator. (An easy
way to construct the noisy signal, is, if $x$ is the 1 MHz samples generated in part e), then form the noisy version as $x_{\text{noisy}} = x + \text{sig} \ast \text{randn}(1, \text{length}(x);:)$. Given the noisy signal, then do the bandpass sampling in two ways.

i. Ignore the noise (based on the argument that the noise standard deviation, $\text{sig} = 20$, is much less than the peak signal amplitude of 400) and proceed with the bandpass sampling as in part e). Plot the signal spectrum (apply the $\text{fft}$ to the bandpass samples). Then from the plot (of the magnitude, in dB) measure the bandwidth at the frequencies when the magnitude is -30 dB from the peak. Compare to the result in part e).

ii. Before bandpass sampling, apply a bandpass digital filter. (E.g., use one generated in Matlab using the instructions

\[ h = \text{fir1}(100, [100, 110]/500, '\text{bandpass}'); \ y = \text{conv}(x_{\text{noisy}}, h); : \]

Apply the same bandpass sampling to the filtered signal, plot the resulting spectrum (again, use the $\text{fft}$), and compute the effective signal bandwidth. Explain qualitatively the difference between using the bandpass filtering and not using the bandpass filtering (when there is out-of-band noise of other signal content).

2. Let $x(n) = (1, 1, -1, -2, 1)$ and $h(n) = (2, 2, 1, 1)$ beginning with sample $n = 0$. Consider the signal processing diagrammed below.

(a) Find $\tilde{y}(n)$ if $N = 6$.
(b) Find $\tilde{y}(n)$ if $N = 10$.
(c) Let $y(n) = x(n) \ast h(n)$, the linear convolution of $x(n)$ and $h(n)$. What is the smallest value of $N$ such that $\tilde{y}(n) = y(n)$, over the range of $n$ for which $y(n)$ is non-zero?

3. Design a comb filter to eliminate all even harmonics of 60 Hz, (i.e., 0, 120, 240, ... Hz) up to 1200 Hz. Assume a sampling rate of 2,400 Hz.
4. Let signal \( x(n) \), \( n = 0, \ldots, N - 1 \) have \( N \)-point DFT \( X(k) \), \( k = 0, \ldots, N - 1 \), with \( N \) even. Find, in terms of \( X(k) \) or \( x(n) \), as appropriate, the DFT (or IDFT) of the following.

a) \( x_1(n) = \begin{cases} x(n/2), & n \text{ even;} \\ 0, & n \text{ odd.} \end{cases} \) Find \( X_1(k) \), the \( 2N \)-point DFT of \( x_1(n) \).

b) \( x_2(n) = (-1)^n x(n) \). Find \( X_2(k) \), the \( N \)-point DFT of \( x_2(n) \).

c) \( x_3(n) = x(n - N/2) \). Find \( X_3(k) \), the \( N \)-point DFT of \( x_3(n) \).

d) \( X_4(k) = \begin{cases} X(k), & k = 0, \ldots, N/2 - 1; \\ 1/2 X(k/2), & k = N/2; \\ 0, & N/2 + 1 \leq k < 3N/2; \\ 1/2 X(k/2), & k = 3N/2; \\ X(k - N), & 3N/2 < k \leq 2N - 1. \end{cases} \) Find \( x_4(n) \), the \( 2N \)-point IDFT of \( X_4(k) \).

e) \( X_5(k) = \begin{cases} X(k/2), & k \text{ even;} \\ 0, & k \text{ odd.} \end{cases} \) Find \( x_5(n) \), the \( 2N \)-point IDFT of \( X_5(k) \).

f) \( x_6(n) = \begin{cases} x(n), & 0 \leq n < 8; \\ x(n) = 0, & 8 \leq n < 16. \end{cases} \) Find \( X_6(k) \), the \( 2N \)-point DFT of \( x_6(n) \).

5. A length-8 signal \( x(n) = [2, 1, 0, -1, -1, -1, 0, 1] \) has 8-point DFT \( X(k) = [1, 5.8284, 1, 0.1716, 0, 0.1716, 1, 5.8284] \), as shown as the top left and top right plots in the figure below. In the figure, correctly identify the signal

i) \( x_1(n) = x(n - 4) \) and its 8-point DFT \( X_1(k) \); and ii) \( x_2(n) = (-1)^n x(n) \) and its 8-point DFT \( X_2(k) \).
6. Consider now signal modifications due to zero padding, zero insertion, and signal repetition. The top left and right plots in the figure below correspond to the length-8 signal $x(n)$ and its 8-point DFT $X(k)$. Identify in the figure the plots of the length-16 sequences and their respective 16-point DFTs i) $y(n)=[x(0),\ldots,x(7),x(0),\ldots,x(7)]$ and $Y(k)$; ii) $y_1(n)=[x(0),\ 0,x(1),\ 0,\ldots,x(7),\ 0]$ and $Y_1(k)$; and iii) $y_2(n)=[x(0),\ldots,x(7),\ 0,\ldots,\ 0]$ and $|Y_2(k)|$ (we consider the magnitude plot of the zero-padding case because the DFT in complex-valued).