1. Text, problem 1.9.


3. Text, problem 1.17.

4. Determine whether or not each of the following systems is i) causal, ii) memoryless, iii) linear, iv) time-invariant, and v) BIBO stable.
   
   (a) \( y(n) = (x(n) - x(n - 2))^2 \)
   
   (b) \( y(n) = \sum_{k=-5}^{5} \frac{1}{11} x(n - k) \)

   (c) \( y(n) = \sum_{k=-5}^{n} (\frac{1}{2})^{|k|} x(n - k) \)

5. An \( L \)-level quantizer has input \( X \), output \( Y \), and error \( q = X - Y \). Assume that \( X, Y, \) and \( q \) are random variables satisfying \( E[X] = 0, E[Y] = 0 \), and hence that \( E[q] = 0 \), that \( E[X^2] = \sigma_X^2 \), \( E[Y^2] = \sigma_Y^2 \), \( E[q^2] = \sigma_q^2 \), and that \( Y \) and \( q \) are uncorrelated, i.e., that \( E[Yq] = 0 \).
   
   (a) Evaluate \( E[X^2] = E[(Y + q)^2] \) and conclude that \( E[Y^2] = E[X^2] - E[q^2] \).

   (b) Evaluate \( E[Y^2] = E[(X - q)^2] \) and conclude that \( X \) and \( q \) must be positively correlated. Determine \( E[Xq] \) in terms of \( \sigma_q^2 \).

Define the “boxcar” function as

\[
p_T(t) = \begin{cases} 
1, & \text{if } |t| < T \\
\frac{1}{2}, & \text{if } |t| = T \\
0, & \text{otherwise}
\end{cases}
\]

and the sinc function as

\[
sinc(t) = \begin{cases} 
1, & \text{if } t = 0 \\
\frac{\sin(\pi t)}{\pi t}, & \text{otherwise}
\end{cases}
\]

Several Fourier transform pairs are listed below.

a. \( p_T(t) \leftrightarrow 2T \text{sinc}(2fT) \)

b. \( 2W \text{sinc}(2Wt) \leftrightarrow P_W(f) \)

c. \( e^{j2\pi f_1 t} \leftrightarrow \delta(f - f_1) \)
d. \( \delta(t - t_1) \leftrightarrow e^{-j2\pi ft_1} \)

e. \( \cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)] \)

f. \( \sin(2\pi f_0 t) \leftrightarrow \frac{1}{2} [\delta(f - f_0) - \delta(f + f_0)] \)

6. Let \( x(t) = 10\text{sinc}(100t) \cos(2\pi 300t) \).

   (a) Find \( X(f) \) and sketch it.

   (b) From the sampling theorem, what is the minimum sampling rate required to sample \( x(t) \) and recover the signal from its samples without aliasing distortion? For the remainder of the problem, let the sampling rate be \( (1/T) = 250 \text{ Hz} \).

   (c) Sketch the sum-of-translates spectrum at the output of the sampler.

   (d) If an ideal low-pass filter with cutoff frequency \( 1/(2T) = 125 \text{ Hz} \) is used to reconstruct an analog signal \( y(t) \) from the samples of \( x(t) \), determine the signal \( y(t) \).

   (e) Determine a suitable bandpass filter that can be used to reconstruct \( x(t) \) from its samples taken at sampling rate 250 Hz.

7. A sampling/reconstruction system is diagrammed below. Assume for the problem that the sampling rate is \( (1/T) = 10,000 \text{ Hz} \).

   (a) Let \( x(t) = \sin(2\pi 3000t) \). Sketch the spectrum at the output of the sampler, and find \( y(t) \).

   (b) Repeat part a) with \( x(t) = \sin(2\pi 7000t) \).

   (c) Repeat part a) with \( x(t) = \sin(2\pi 13000t) \).