1. (Text, problem 1.9.) Consider the following periodic signal, \( x_a(t) = 1 + \cos(10\pi t) \).

(a) Compute the magnitude spectrum of \( x_a(t) \).

(b) Suppose \( x_a(t) \) is sampled with a sampling frequency of \( F_s = 8 \text{ Hz} \). Sketch the magnitude spectra of \( x_a(t) \) and the sampled signal, \( \hat{x}_a(t) \).

(c) Does aliasing occur when \( x_a(t) \) is sampled at the rate \( F_s = 8 \text{ Hz} \)? What is the folding frequency in this case?

(d) Find a range of values for the sampling interval, \( T \), that ensures that aliasing does not occur.

(e) Assuming \( F_s = 8 \text{ Hz} \), find an alternative lower-frequency signal, \( x_b(t) \), that has the same set of samples as \( x_a(t) \).

2. (Text, problem 1.13.) There are special circumstances where it is possible to reconstruct a signal from its samples, even when the sampling rate is less than twice the bandwidth. To see this, consider a signal, \( x_a(t) \), whose spectrum is \( X_a(f) \), with a "hole in it" between 100 and 400 Hz, as shown in the figure below.

![Spectrum](image)

(a) What is the bandwidth of the signal \( x_a(t) \)?

(b) Suppose the sampling rate is \( F_s = 750 \text{ Hz} \). Sketch the spectrum of the sampled signal, \( \hat{x}_a(t) \).

(c) Show that \( x_a(t) \) can be reconstructed from \( \hat{x}_a(t) \) by finding an idealized reconstruction filter with input \( \hat{x}_a(t) \) and output \( x_a(t) \). Sketch the magnitude response of the reconstruction filter.

(d) For what range of sampling frequencies below \( 2F_s \) can the signal be reconstructed from the samples using the type of reconstruction filter from part c)?

3. (Text, problem 1.17.) Suppose a bipolar ADC is used with a precision of \( N = 12 \text{ bits} \) and a reference voltage of \( V_r = 10 \text{ volts} \).
(a) What is the quantization level, \( q \)?

(b) What is the maximum value of the magnitude of the quantization noise, assuming the ADC input-output characteristics are offset by \( q = 2 \)? (As shown in Fig. 1.37 in the text, Edition 3, or Fig. 1.34 in Edition 1.)

(c) What is the average power of the quantization noise?

4. Determine whether or not each of the following systems is i) causal, ii) memoryless, iii) linear, iv) time-invariant, and v) BIBO stable.

(a) \( y(n) = (x(n) - x(n - 2))^2 \)

(b) \( y(n) = \sum_{k=-5}^{5} \frac{1}{11} x(n - k) \)

(c) \( y(n) = \sum_{k=-5}^{5} (\frac{1}{2})^{|k|} x(n - k) \)

5. An \( L \)-level quantizer has input \( X \), output \( Y \), and error \( q = X - Y \). Assume that \( X, Y, \) and \( q \) are random variables satisfying \( E[X] = 0 \), \( E[Y] = 0 \), and hence that \( E[q] = 0 \), that \( E[X^2] = \sigma_X^2 \), \( E[Y^2] = \sigma_Y^2 \), \( E[q^2] = \sigma_q^2 \), and that \( Y \) and \( q \) are uncorrelated, i.e., that \( E[Yq] = 0 \).

(a) Evaluate \( E[X^2] = E[(Y + q)^2] \) and conclude that \( E[Y^2] = E[X^2] - E[q^2] \).

(b) Evaluate \( E[Y^2] = E[(X - q)^2] \) and conclude that \( X \) and \( q \) must be positively correlated. Determine \( E[Xq] \) in terms of \( \sigma_q^2 \).

Define the “boxcar” function as

\[
p_T(t) = \begin{cases} 
1, & \text{if } |t| < T \\
\frac{1}{T}, & \text{if } |t| = T \\
0, & \text{otherwise}
\end{cases}
\]

and the sinc function as

\[
sinc(t) = \begin{cases} 
1, & \text{if } t = 0 \\
\frac{\sin(\pi t)}{\pi t}, & \text{otherwise}
\end{cases}
\]

Several Fourier transform pairs are listed below.

a. \( p_T(t) \leftrightarrow 2T \text{sinc}(2ft) \)

b. \( 2W \text{sinc}(2Wt) \leftrightarrow P_W(f) \)

c. \( e^{j2\pi ft} \leftrightarrow \delta(f - f_1) \)

d. \( \delta(t - t_1) \leftrightarrow e^{-j2\pi ft_1} \)

e. \( \cos(2\pi ft) \leftrightarrow \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)] \)

f. \( \sin(2\pi ft) \leftrightarrow \frac{j}{2}[\delta(f - f_0) - \delta(f + f_0)] \)
6. Let \( x(t) = 10 \text{sinc}(100t) \cos(2\pi 300t) \).

   (a) Find \( X(f) \) and sketch it.

   (b) From the sampling theorem, what is the minimum sampling rate required to sample \( x(t) \) and recover the signal from its samples without aliasing distortion? For the remainder of the problem, let the sampling rate be \((1/T) = 250 \text{ Hz}\).

   (c) Sketch the sum-of-translates spectrum at the output of the sampler.

   (d) If an ideal low-pass filter with cutoff frequency \((1/(2T)) = 125 \text{ Hz}\) is used to reconstruct an analog signal \( y(t) \) from the samples of \( x(t) \), determine the signal \( y(t) \).

   (e) Determine a suitable \textit{bandpass} filter that can be used to reconstruct \( x(t) \) from its samples taken at sampling rate 250 Hz.

7. A sampling/reconstruction system is diagrammed below. Assume for the problem that the sampling rate is \((1/T) = 10,000 \text{ Hz}\).

   (a) Let \( x(t) = \sin(2\pi 3000t) \). Sketch the spectrum at the output of the sampler, and find \( y(t) \).

   (b) Repeat part a) with \( x(t) = \sin(2\pi 7000t) \).

   (c) Repeat part a) with \( x(t) = \sin(2\pi 13000t) \).