Optimal Partitioning of Multicast Receivers

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Outline of Presentation

- Motivation
- Optimal Receiver Partitioning
- Max-min Fair Rate Optimal Partitioning
- Experimental Evaluations
- Conclusion
Why Optimal Partition?

- Heterogeneous receiver capacities
  - Receiver host restrictions
  - Network path: Modem, ISDN, Cable
    Modem, LAN
How to determine the sending rate(s)?

- **Single-rate**
  - The lowest rate
  - To maximize inter-receiver fairness

- **Multi-rate**
  - Q1: How many groups?
  - Q2: How to determine the rates?
Q1: How many groups?

- As many as receivers
- A fixed number of groups, e.g., 4 groups
- The more groups, the higher
  - the sender overhead to encode
  - the network overhead to keep the states
  - the receiver overhead to decode
- Our result: 4-5 groups for majority of benefits
Q2: How to determine the rates?

- **Static**
  - Independent of receiver capacities
  - Determined by encoding scheme

- **Dynamic**
  1. Heuristics
  2. Our solution: dynamic programming to find an optimal solution
Answer to Q2: Optimal Receiver Partition

**Terminology**
- Isolated rate $r_i$
- Receiver utility function $u(r, g)$
- Group utility
  $$ U(G, g) = \sum_{i \in G} u(r_i, g) $$

<table>
<thead>
<tr>
<th>$r_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(r_i, g)$</td>
<td>1.0</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

$$ U(G, g) = 1.0 + 0.5 + 0.3 = 1.8 $$
Receiver Utility Function

- $u(r, g)$

Properties
- Non-decreasing when $r$ and $g$ approach to each other.
- Maximum when $r = g$.

Examples

\[ u(r, g) = \frac{\min(r, g)}{\max(r, g)} \]

\[ u(r, g) = \min(r, g) \]
Optimal Partition

■ Session utility

\[ V (\{(G_1, g_1), \emptyset, (G_K, g_K)\}) = \sum_{k=1}^{K} U(G_k, g_k) \]

■ Optimal partition
  – Maximizes the session utility.

\[
\begin{array}{c|ccc}
 r_i & 1 & 2 & 3 \\
\hline
 u(r_i, g_1) & 1.0 \quad 1.0 \quad 0.7 \\
 u(r_i, g_2) \\
\end{array}
\]

\[
U(G_1, g_1) = 1.0 \\
U(G_2, g_2) = 1.0 + 0.7 = 1.7 \\
V = 1.0 + 1.7 = 2.7
\]
Finding the Optimal Partition

- Ordered partition
  - If \( i \in G_k \) and \( j \in G_{k+1} \), then \( r_i \leq r_j \).

- There exists an ordered partition that is optimal.
Finding the Optimal Partition

- Dynamic programming algorithm
  - Finds the ordered optimal partition

\[ V^*(i, m) = \max_{1 \leq j < i} \left( V^*(j, m-1) + U^*(\{j+1, \emptyset, i\}) \right) \]
Answer to Q1: Number of Groups

4 groups for 80% of the maximum.

\[ u(r, g) = \frac{\min(r, g)}{\max(r, g)} \]

\[ u(r, g) = \min(r, g) \]
Collecting Isolated Rates

- Max-min fair rates as isolated rates
- Aggregation

![Diagram showing the aggregation of rate intervals]
Experimental Evaluations

Aggregation error < 3% with 4 intervals.
Conclusion

- Heterogeneous receiver capacities.
- Determine the optimal receiver partition in multi-rate multicasts.
- Achieve 80% of the maximal utility.
- Future work
  - Extension to other network fairness.
The End

Question?