Problem 1

a. Please choose a control system that interests you. For the example that you choose, please describe what the following pieces of the control system are: plant, actuator, process, plant input, plant output, sensor, controller, and reference signal.
b. Is the control system that you chose in the first part of the problem 1) manual or automatic, 2) a regulator or servo, and 3) open-loop or closed-loop?

Problem 2

a. What is the difference between a regulator and a servo system? Which do you think is harder to build?
b. What are advantages and drawbacks of manual control systems vs. automatic ones?
c. Please give an example of a closed-loop automatic servo system.

Problem 3

The input $u(t)$ and output $y(t)$ of a plant that you would like to control are related by $y(t) = 7u(t) + w(t)$, where $w(t)$ is an additive disturbance at the output that is at most one unit in magnitude (i.e., $-1 \leq w(t) \leq 1$ for all times $t$). You would like to build a controller so that the output follows a constant reference signal $r(t) = \bar{r}$, where $-10 \leq \bar{r} \leq 10$. You will consider both open-loop and closed-loop control in this problem.

a. Please draw a block diagram for the plant.
b. Please build an open-loop controller that sets the output to $\bar{r}$, assuming that the disturbance is ignored. Please show your controller both as an equation and a block diagram.
c. Say that you use the open-loop controller from part b, but now a disturbance is present. Please find the maximum possible magnitude of the error in the output. Also, for the particular disturbance $u(t) = \sin(t)$, what is the root-mean-square (RMS) error.
For the rest of the problem, consider feedback control of the plant. In particular, say we use a controller of the form $u(t) = K (\bar{r} - y(t))$, where $K$ is a gain constant to be designed.

d. Please draw the full control system when the closed-loop controller is used.
e. Please find expressions for output, plant input, and error, in terms of $\bar{r}$, $w(t)$, and $K$.
f. Please design $K$ so that the absolute value of the error is at most 0.1.
g. In general, making the gain bigger would make the error smaller and smaller in steady-state. What limits the size of the gain that we can use?
h. Since you built the feedback controller, you have access to the controller's input and output (which are the error signal and the plant input, respectively). Using this information, can you compute the disturbance signal $w(t)$?

**Problem 4**

a. Please explain what a model is.
b. You have already done a lot of modeling work in your previous classes. In what senses is modeling for controller design different?
c. Please list the three common forms for linear time-invariant differential equations.
d. Please write the differential equation $\frac{d^3 y}{dt^3} + 2 \frac{d^2 y}{dt} + 8y = u$ in state-space form and transfer-function form.
e. Please write the differential equation $\frac{d^3 y}{dt^3} + 2 \frac{d^2 y}{dt} + 8y = u + 6 \frac{du}{dt}$ in state-space form and transfer-function form.
EE 489, Homework 2

Problem 1

Consider the system consisting of objects and springs shown below. Here, the two objects' masses are assumed unknown, and are called $m_1$ and $m_2$. The following forces act on the objects: a input force $u(t)$ acts on Object 1, three springs (with spring constants shown below) exert forces on the objects, and friction acts on both objects as shown below. We note that the springs are at their nominal lengths (i.e., they exert no force) when the masses are at their nominal positions ($x_1 = 0$ and $x_2 = 0$). The output $y(t)$ that you are interested in tracking is the position $x_2(t)$ of object 2. Please leave all your answers to this problem in terms of the masses $m_1$ and $m_2$.

\[ \text{All spring constants are 1, both friction constants are 1} \]

\[ x_2 = y \]

a. Please write a pair of differential equations for the positions of the two objects, $x_1(t)$ and $x_2(t)$.

b. The input $u(t)$ and the output $y(t) = x_2(t)$ are related by a fourth-order differential equation. Please write the differential equation in 1) standard form, 2) state-space form, and 3) transfer function form.

c. If we drive the system with a constant input for $t \geq 0$, would you expect the position of the objects to settle to a steady-state? If yes, please find the steady-state locations of the objects for the input $u(t) = 1$, $t \geq 0$. 

d. Do the masses of the objects play any role in the steady-state of the system? How about in its dynamics?

e. If you decrease the spring constant for the rightmost spring (while leaving all other parameters identical), how will the steady-state positions of the two masses change?

**Problem 2**

Please consider a pendulum consisting of two point masses on a single hard rod. The torques acting on the pendulum (as well as the gravitational force) are diagrammed below:

![Diagram of a pendulum with masses and forces labeled]

a. Please write a nonlinear differential equation for the pendulum’s angle $\theta$. (You may leave your answer in terms of the unknown constants $m_1$, $m_2$, $l_1$, and $l_2$).

b. For the constant input $u(t) = \text{overline}{u}$ for $t \geq 0$, what is the steady-state angle $\text{theta}$ of the pendulum. (Again, you may leave your answer in terms of the unknown constants and the input level.)

**Problem 3**

Consider the following system, written in state-space form:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-2 & 1 \\
1 & -2
\end{bmatrix} \begin{bmatrix}
\chi_1 \\
\chi_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

\[
y = \begin{bmatrix}
1 & 0
\end{bmatrix} \begin{bmatrix}
\chi_1 \\
\chi_2
\end{bmatrix}
\]
a. Please write the differential equation in the other two typical forms.
b. Can you find an alternate state-space representation for the same system?

**Problem 4**

Consider the series RLC circuit drawn below, where the input $u(t)$ is the source current and the output $y(t)$ is the current through the resistor. Please write the differential equation for this system in the three typical forms. Please leave your answer in terms of $R$, $L$, and $C$. 

![RLC Circuit Diagram](image-url)
Problem 1

Consider the DC motor model that we developed in class.

a. Say that we drive the motor with a constant input \( u(t) = \bar{u} \). Please find the steady-state angular speed \( \bar{y} \) of the motor, in terms of \( \bar{u} \) and the motor parameters \( (J, K_t, b, R_a, L_a, \text{ and } K_e) \).

b. Look at your answer to part a. Are there motor parameters that do not impact the steady-state? If yes, why does this make sense?

c. Consider increasing each of the motor’s parameters that do impact the steady-state. In which cases will the steady-state speed increase? How about decrease?

d. Let’s say that the armature resistance is doubled. Can another parameter be changed systematically, to maintain the original steady-state speed?

e. If the resistance \( R \) and/or the friction \( b \) could be set o 0, would the motor be able to accelerate forever (according to the DC motor model)?

f. By changing one motor parameter, can you
   1) decrease the steady-state rotor speed while
   2) increasing the steady-state armature current?

Problem 2

Consider a system whose input and output are related by
\[
\begin{align*}
y(t) &= 2u(t) + 3\frac{du}{dt} + 4 \int_0^t u(\sigma)d\sigma.
\end{align*}
\]

Notice that the output here is the linear combination of the input, its derivative, and its integral. Such relationships are common, for instance, in PID control.

a. Please draw a circuit with voltage input \( u(t) \) and voltage output \( y(t) \) that have the above relationship.

b. Can the differential-equation input-output relationship considered in this problem be placed in standard form?
Problem 3

Consider the nonlinear system with state equation \( \dot{x} = e^{-x} - (x + 1)u \) and output equation \( y = \sqrt{x + u} \).

a. For the nominal input \( u(t) = \bar{u} = 1 \), please show that \( x(t) = \bar{x} = 0 \) and \( y(t) = \bar{y} = 1 \) are equilibrium solutions for the system.

b. Please linearize the differential equation around the equilibrium point from part a.

c. Please find the transfer function from the input to the output of the linearized equation.

Problem 4

Please consider the system of \( n \) tanks diagrammed below. Please find the transfer function for this system.

\[ A = 1, \ p = 1, \ g = 10, \]

Flow rate out = \[ \frac{1}{10} \text{(press. diff.)} \]

for each tank.
Homework 4

Problem 1

Consider the linear system \( \frac{d^2 y}{dt^2} + 7 \frac{dy}{dt} + 6y = \frac{dy}{dt} + 2u, \ t \geq 0. \)

a. Please find the impulse response of this system.
b. Please explain in words what an impulse response is.
c. Please find the zero-state response for the following inputs: i) \( u(t) = e^{-t}, \ t \geq 0; \) ii) \( u(t) = 1, \ 0 \leq t \leq 1 \) and 0 otherwise; iii) \( u(t) = t, \ 0 \leq t \leq 1, \ u(t) = 2 - t, \ 1 \leq t \leq 2 \) and \( u(t) = 0 \) otherwise.
d. Please find the zero input response, when the initial conditions are \( y(0^-) = 1 \) and \( \frac{dy}{dt}(0^-) = 4. \)
e. Please find the response \( y(t) \) when the input is \( u(t) = 1 + e^{-t}, \ t \geq 0, \) and the initial conditions are \( y(0^-) = 2 \) and \( \frac{dy}{dt}(0^-) = 8. \)

Problem 2

Please take the inverse Laplace transforms of the following \( s \)-domain signals:

a. \( Y(s) = \frac{1}{(s+4)^2(s+2)} \)
b. \( Y(s) = \frac{s+7}{s^2+4s+13} \)
c. \( Y(s) = \frac{s+7}{(s+2)^2-3^2} \)
d. \( Y(s) = \frac{2}{s^2}(1-e^{-s} - e^{-2s}) \)
e. \( Y(s) = \frac{(s+2)^2+10}{(s+2)^2+3^2} \)

Problem 3

A proper linear time-invariant system has shown up on your desk at work. You have no idea how it behaves, so you decide to drive the system with a particular input, and measure the response. When you put in the input \( u(t) = e^{-t}, \ t \geq 0, \) you find that \( y(t) = 1 - e^{-t}, \ t \geq 0. \) You may assume that the system is initially relaxed (i.e., the initial conditions are 0). Please answer the following questions.

a. Find the transfer function from \( u \) to \( y. \)
b. Please find the impulse response \( h(t). \)
c. What is the output \( y(t) \) when the input is \( u(t) = e^{-2t}. \)
d. What is the output \( y(t) \) when the input is \( u(t) = \sqrt{t+1}. \)
Problem 4

You are working with a system that is known to be governed by a first-order linear time-invariant differential equation, but is otherwise unknown. When you set the initial condition of the system to $y(0^-) = 1$ and put in a particular input $\bar{u}(t)$, you find that the output at time 7 is $y(7) = 4$. When you set the initial condition to $y(0^-) = 3$ and put in the input $5\bar{u}(t)$, you find that $y(7) = 25$. If you set the initial condition to $y(0^-) = 2$ and put in the input $6\bar{u}(t)$, what is $y(7)$? (Hint: you don’t actually have to figure out what the system is.)
Homework 5

Problem 1: Scaling

For some strange reason, you decide to develop a dynamic model for the mass of an elephant, with mass measured in atomic mass units (elementary particles). You also choose to use seconds as the time units for the model. After several years of study, you derive the following model: 
\[
\dddot{y} + 10^{-5}\ddot{y} + 10^{-10}y = 10^{-2}\cdot10^{2.5}u
\]
where \( y \) is the elephant mass and \( u \) is the food input (in kilograms). Unfortunately, you are unable to simulate this differential equation because of the extreme coefficient values, and your whole elephant-mass study seems about to fail 😞. Please use magnitude- and time-scaling to rewrite the differential equation in a more usable form.

Problem 2: Solving Linear Systems

Consider the system 
\[
\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{du}{dt} + u
\]
Please find the following responses:

a. Find the impulse response of the system.
b. Find the output \( y(t) \), \( t \geq 0 \), when the input is a unit step and the initial conditions are \( y(0^-) = \frac{dy}{dt}(0^-) = 2 \).
c. Find \( y(t) \), \( t \geq 0 \), when the input is a unit pulse (\( u(t) = 1 \), \( 0 \leq t \leq 1 \) and \( u(t) = 0 \) otherwise) and the initial conditions are \( y(0^-) = \frac{dy}{dt}(0^-) = 2 \).
d. Find the zero-state response of the system (for \( t \geq 0 \)) when the input is \( u(t) = e^{-2t}, t \geq 0 \).
e. Find the zero-state response (for \( t \geq 0 \)) when the input is \( u(t) = e^{-2t}\sin(3t) \).
f. You drive the system with an unknown input and find that the response is \( y(t) = -3e^{-3t} + 4e^{-4t} \), for \( t \geq 0 \). Please find the input \( u(t) \) for \( t \geq 0 \). (Assume zero initial conditions for this part.)
Problem 3: Finding Heat Flow

\[
\begin{array}{c|c}
T_1(0^-) = 20^\circ & T_2(0^-) = 50^\circ \\
T_1(t) & T_2(t) \\
\xi = 1 & \kappa = 1 \\
\end{array}
\]

- Insulator: no heat flow

a. Find the temperature of each room for all times \( t \geq 0 \).
b. Find the average temperature of the two rooms, for all times \( t \geq 0 \).

Problem 4: Using Linearity

You drive a linear time-invariant system with a particular input from a particular initial condition, and find that the output at time 7 is \( y(7) = 5 \). If you double the input and triple the initial condition, you find that \( y(7) = 14 \). If you were to instead halve the original input and divide the original initial condition by 3, what would the output at time 7 equal?

Problem 5: Stability

For each of the following systems, please answer the following questions: 1) Is the system BIBO stable, not BIBO stable, or indeterminate (meaning that stability cannot be conclusively determined)? 2) Is the system internally stable, internally marginally stable, internally unstable, or undecidable? (Please assume throughout that internal stability can be determined from the input-output relationship of the system.)

a. A system with output response \( y(t) = e^{0.5t} + (u(t))^2 \).
b. A system with output response \( y(t) = \int_{0}^{t} u(\sigma) \, d\sigma \).
c. \( \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = \frac{dy}{dt} - u \)
d. \( \frac{d^2y}{dt^2} - 5 \frac{dy}{dt} - 6y = u \)
e. \( \frac{d^2y}{dt^2} - 5 \frac{dy}{dt} - 6y = \frac{dy}{dt} - 6u \)
f. \( \frac{d^2y}{dt^2} + y = u \).
g. \( \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + y = u \).
h. \( \frac{d^2y}{dt^2} + 17 \frac{dy}{dt} + 16 \frac{dy}{dt} - \frac{dy}{dt} + 6y = u \)
i. \( \frac{d^2y}{dt^2} + K \frac{dy}{dt} + K \frac{dy}{dt} + K \frac{dy}{dt} + Ky = u \). (Please state your answer in terms of \( K \).) j. A system for which the output is \( y(t) = 2e^{t} \) when the input is \( u(t) = 2e^{-t} \) and the initial condition is 0.
Problem 1: Linear System Performance

Consider the unit-step response of the system with transfer function $H(s) = \frac{25}{s^2 + 6s + 25}$.

a. Is the system BIBO stable?

b. What is the output of the system in steady-state?

c. What are the rise time, settling time, peak time, and overshoot of this system?

d. If we cascade this system with another system that has transfer function $H_2(s) = \frac{20}{(s+4)^2+s^2}$ (see below), what are the steady-state output, rise time, settling time, peak time, and overshoot (of the unit-step response) of the cascaded system? (You may use the worst-pole approximation.)

![Cascaded System Diagram]

Problem 2: Performance, General System

We need for a particular LTI system to 1) be BIBO stable, 2) have a rise time of less than 0.9, 3) have an overshoot of less than 10%, and 4) have a settling time of less than 4.6. Please sketch the region in the complex plane where the poles of the system may lie. (You may use the worst-pole approximation.)

Problem 3: Testing the Approximation

Consider the system $H(s) = \frac{s+10}{(s+5)(s+20)}$.

a. Based on the “worst-pole” assumption, what would you expect the rise time, settling time, and overshoot of this system to equal? (Notice that your answer essentially won’t depend on the location of the zero of $H(s)$.)

b. Experimentally find the rise time, settling time, and overshoot for several values of a. I recommend using Matlab to do this; you can use the command step([1,a],[1,22,40]). How do the performance characteristics depend on the location of the zero?
Problem 4: Analyzing Block Diagrams

Please find the transfer function from $R$ to $Y$ for the block diagrams in Problems 3.20b and 3.21 in your text. (These two block diagrams are also shown on an attached page.)

Problem 5: Unity-Feedback-Gain Systems

a. Please find the transfer function from $R$ to $Y$ and from $R$ to $E$ in the block diagram shown above.
b. For what gains $K$ is the system bounded-input bounded-output (BIBO) stable.
c. What is the system type for reference tracking? What is the steady-state tracking error when the input is $u(t) = t^n$ ($n = 0, 1, 2, \ldots$)?

Problem 6

We are often concerned with whether a system can reject polynomial disturbances of different degrees. For any control system in the standard unity-feedback-gain configuration, we find that the closed-loop system can totally reject disturbances up to a certain degree (i.e., the output in response to the disturbance is 0 in steady-state). The system can reject disturbances of the next-higher degree up to a finite value, and cannot reject disturbances of even higher degree. The highest degree of disturbance that can be rejected up to a finite value is known as the system type for disturbance rejection.

a. Prove that the system type for disturbance rejection is equal to the number of poles of the controller $D(s)$ at the origin. (You may assume the plant/controller have no zeros at the origin.)
b. Consider control of the plant $G(s) = \frac{1}{s(s+3)}$ using the controller $D(s) = \frac{1}{s} + 1$ (in the standard unity-feedback-gain configuration). Show that the closed-loop is stable. Also, find the steady-state output of the closed-loop system when it is affected by the following disturbances (assuming no reference signal and sensor noise): i) $w(t) = 1$, ii) $w(t) = 2t$, iii) $w(t) = t^3$. What is this system's type for disturbance rejection? How about for reference tracking?

Problem 7

We are interested in building a feedback controller (in the standard unity-feedback-gain configuration) for the plant $G(s) = \frac{2}{s(4s+2)}$.

a. Let's say that we use the controller $D(s) = k(s^2 + 3 + s)$. What is the steady-state error, when this closed-loop system tries to track the reference signal $r(t) = t^3 + 2t + 1$ and is affected by the disturbance $w(t) = 4t$? Please leave your answer in terms of the gain $k$. 

2
b. Now let's say we try to design a controller for this plant, so that the closed-loop system can track a cubic reference signal with zero error, and can reject a cubic disturbance up to a finite error. What is the minimum number of poles at the origin required of the controller?
Problem 1

In this problem, you will consider control of the plant \( G(s) = \frac{\frac{1}{s^2+4s+3}}{s^3+4s+3} \), using a feedback controller in the standard unity-feedback-gain configuration.

a. First, consider using the proportional controller \( D(s) = k \), and answer the following questions:
   i) For \( k = 0.5, 5, \) and \( 50 \), find the steady-state tracking error for a unit-step reference.
   ii) Using Matlab, plot the response of the closed-loop system to a unit-step reference, for \( k = 0.5, 5, \) and \( 50 \). Also, please plot the error signal and the actuation signal (the input into the plant), for \( k = 5 \).
   iii) Discuss how the steady-state error, rise time, and overshoot of the closed-loop system's step response depend on the gain \( k \). Is it possible to choose \( k \) so that the overshoot and steady-state error are both small?

b. Next, consider controlling this plant using the PI controller \( k(1 + \frac{1}{s}) \), and answer the following questions:
   i) For what gains \( k \) is the closed-loop system stable?
   ii) Find the steady-state tracking error for a unit-step reference, in terms of \( k \).
   iii) By simulating the response for several \( k \), figure out how the rise time and overshoot depend on \( k \).
   iv) Is it possible to make both the steady-state error and overshoot small, when a PI controller is used?

Problem 2

Please show that each of the following problems can be viewed as a root locus problem, and find the root-locus function \( L(s) \) in each case:

a. A plot of the roots of \( s^3 + cs^2 + cs + 1 \), as \( c \) varies from 0 to \( \infty \).

b. A plot of the roots of \( cs^3 + s^2 + s + 1 \), as \( c \) varies from 0 to \( \infty \).

c. A plot of the roots of \( s^3 + cs^2 + s + 1 \) as \( c \) varies from 1 to \( \infty \).

d. For the plant \( G(s) = \frac{1}{s(s+3)} \) and a proportional controller (in the unity-feedback-gain configuration), a plot of the closed-loop system's poles for all gains that make the closed loop system stable.

Problem 3

For each given \( L(s) \), please draw the corresponding root locus by hand, making sure to explain how each of the six root-locus rules is used (if it is).
a. \( L(s) = \frac{1}{s+3} \)  
b. \( L(s) = \frac{2}{s^2 + s + 2} \)  
c. \( L(s) = \frac{(s+6)(s+1)^2}{s^2 + 6s + 9} \)  
d. \( L(s) = \frac{(s+2)(s+4)}{s^2 + 2s + 4} \)  
e. \( L(s) = \frac{s^2 + 2}{s(s+4)(s+6)(s+8)} \)

(Hint: Rules 5 and 6 are tedious to apply for part e, so please use a calculator or Matlab to find roots as needed.)

**Problem 4**

Please use Matlab to draw the root loci from Problem 2. Please try using both “rlocus” and “rltool/sisotool”...you will need some of these loci later in the homework, so it’s helpful to play with them using the Matlab tool.

**Problem 5**

Please prove Rule 4, Part 1, for drawing root loci.

**Problem 6**

a. Consider proportional control of the plant \( G(s) = \frac{(s+4)}{(s+6)(s+1)^2} \) (in the unity-gain-feedback configuration), and answer the following questions using the appropriate root locus:

i) What is the smallest achievable settling time?

ii) What is the smallest achievable rise time?

iii) If we wish to limit the overshoot to 10%, what are the smallest achievable settling time and rise time? What gain achieves this rise time and settling time? (For finding the gain, please think about how you can do this by hand, in addition to using Matlab.)

iv) If we wish to limit the steady-state error due to a unit step reference to 5%, what is the smallest achievable overshoot?

b. Consider PD control of this plant, using the controller \( D(s) = K(s + 10) \), where \( K \geq 0 \) is a gain parameter. Answer the following questions (by drawing the appropriate root locus in Matlab):

i) What is the smallest achievable settling time?

ii) What is smallest achievable rise time?

iii) If we wish to limit the overshoot to 10%, what are the smallest achievable settling time and rise time?

iv) If we wish to limit the steady-state error due to a unit step reference to 5%, what is the smallest achievable overshoot?

c. Consider PD control of this plant, using the controller \( D(s) = k_p + k_ds \). Find \( k_p \) and \( k_d \) so that \( s = -2 + 4j \) is on the root locus.

d. Consider control of this plant using the controller \( D(s) = \frac{K(s+10)}{s+100} \), where \( K \geq 0 \) is a gain parameter. (This controller is a lead compensator; it is a good approximation for a PD controller, but doesn’t suffer from the noise issues that are typical of PD controllers.) If we wish to limit the overshoot to 10%, what are the smallest achievable settling time and rise time?
1 Problem 1

Consider the plant $G(s)$ with the frequency response shown on the attached page, and answer the following questions.

a. What is the sinusoidal steady-state response of the plant to the input $2 \cos(3t)$?

b. What is the sinusoidal steady-state response to the input $4 \sin(-10t + 40^\circ)$?

c. What is the steady-state output of the plant when it is driven by a unit-step input.

\[ G(s) \] \[ + \] \[ K \] \[ \rightarrow \] \[ \Sigma \] \[ \rightarrow \] \[ Z \]

\[ \text{d. Consider the feed-forward system shown above. Design} \ K \ \text{so that there is a sinusoidal input} \ r \ \text{for which the output} \ z \ \text{is zero in sinusoidal steady-state. What is the frequency of the sinusoidal input for which the output is zero? What is the output} \ z \ \text{when the input} \ r \ \text{is} \ 2 \sin(10t)?} \]

2 Problem 2

Now consider proportional control of the plant from Problem 1 (in the standard unity-feedback-configuration).

a. For what gains $K$ is the closed-loop system stable?

b. As a function of the gain $K$, what is the steady-state error when tracking a unit-step reference signal?

c. For gains $K = 1, 5, 10, \text{and} \ 100$, please find the gain margin, phase margin, and crossover frequency, and then find the rise time, overshoot, and steady-state reference tracking error of the control system.

d. If we wish to limit the overshoot to $30\%$, what is the best achievable steady-state error? e. If we wish to limit the steady-state (unit-step) reference tracking error to $3\%$, what is the minimum achievable overshoot? What gain $K$ achieves this minimum overshoot? What is the rise-time of the closed-loop system in this case?

f. Let’s say we use the gain found in part d. In this case, what are the output and actuation signal of the closed-loop system when the reference signal is $1 + 0.01 \cos(120\pi t)$?

g. Now let’s say we instead use the PD controller $K(1 + \frac{1}{T_d^2})$, where $K$ is the gain that you found in Part e. What are the steady-state tracking error, rise time, and overshoot of your design?

h. When the PD controller from Part g is used, what are the output and actuation signal of the closed-loop system when the reference signal is $1 + 0.01 \cos(120\pi t)$?
Problem 3

Again consider the plant from Problem 1.

a. Design a lead compensator, which limits the unit-step-reference tracking error to 3% and limits the overshoot to 30%. You DO NOT need to iterate on your design.
b. Design a lead compensator for which the closed-loop rise time is less than $\frac{1}{15}$ and the overshoot is less than 30%.
c. If you would like to make sure the rise time, steady-state error, and overshoot requirements are all met, which compensator (the one from part a or the one from part b) should you use?

Problem 4

Consider control of the plant $G(s) = \frac{100}{s(s+2)(s+40)}$ in the standard unity-gain configuration. Specifically, please design a lead compensator for this plant that limits the rise time to 0.36 and limits the overshoot to 25%. Please iterate on your design as needed.